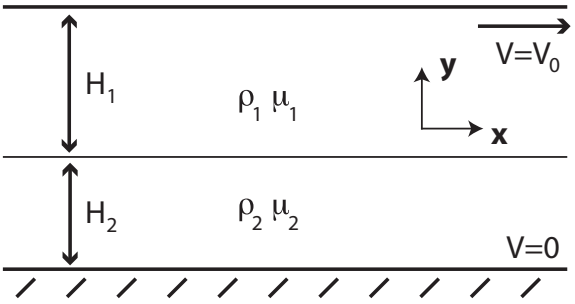
Exercise set 2

Problem 1

Compute the velocity profile for the laminar motion of two layers (thickness $H_{1,2} = 0.05$ m resp. 0.03 m) of incompressible Newtonian fluids (density $\rho_1 < \rho_2$ and viscosity $\mu_{1,2} = 2.28 \cdot 10^{-3}$ $\frac{\text{Ns}}{\text{m}^2}$ resp. 10^{-3} $\frac{\text{Ns}}{\text{m}^2}$) between two plates. One plate is at rest while the other one moves at a velocity $V_0 = 0.05$ $\frac{\text{m}}{\text{s}}$.



Problem 2

Using the Fick law $\mathbf{q} = -D\nabla C$ relating the flux $\mathbf{q}$ of a quantity and the gradient of its concentration C, *compute the total flux of mass of oxygen into a lake* given the concentration profile

$$C(z) = C_{sat} - (C_{sat} - C_l)\,\text{erf}\left(\frac{z}{\delta\sqrt{2}}\right). \tag{1}$$

Here z represents the depth into the lake (taken to be positive) and we assume that the concentration does not depend on the other coordinates (position on the lake). C_{sat} is the saturation oxygen concentration in the water and C_l the oxygen concentration in the body of the lake. δ gives the thickness of the boundary layer over which the concentration goes from the surface value C_{sat} to C_l. Finally the function $\text{erf}(x)$ is defined as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \tag{2}$$